

Engaging Reluctant Problem Solvers

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To support *all* students in meeting the Common Core's Standards for Mathematical Practice, consider the key features of tasks without words.



Think for a moment: What are some different ways that third graders might go about solving the following problem?

Tony owns a pizza parlor. When customers order a pizza, they can choose the type of crust (thick, thin, or original). Customers may also choose one special topping (pepperoni, hamburger, sausage, bacon, or anchovies). How many different types of pizza can Tony make for his customers?

When the task was presented to our class, many students tried to list all possible combinations. One submitted an organized list of the outcomes (see **fig. 1a**). Another chose to figure the total number of combinations without considering what those combinations might be. He simply stated there were five toppings on thick crust, five on thin, and five on original—for a total of fifteen toppings (see

fig. 1b). A third student used a diagram to show the combinations. Drawing circles to represent the types of crust and circles to represent the toppings, he drew lines connecting the toppings to crusts, counting them in groups of three (see **fig. 1c**). As the reader, you may be thinking, *Well, that's cool; but what's the point?* To see the point, let's leave the classroom for just a moment.

With the introduction of the Common Core State Standards (CCSS), teachers must now teach math with an eye on simultaneously meeting the Standards for Mathematical Practice (CCSSI 2010). These Standards “describe varieties of expertise that math educators at all levels should seek to develop in their students” (p. 6). The first Standard, that students “make sense of problems and persevere in solving them,” clearly focuses on problem solving: “Students start by explaining to themselves the meaning of a problem and looking for entry points to its solutions” (p. 6). That is to say, students



ANGELA T. BARLOW

should engage in solving problems for which they have no predetermined way of solving.

Now, let's return to the classroom episode at the beginning of the article. Most readers would argue that each student has provided evidence of understanding the problem and finding an appropriate entry point, either through organized drawings, logical computations, or abstract representations of the problem. Although these students were moving toward meeting the Standard, such was not the case for one of the other students in the class. Jerome began "solving" the problem by neatly writing the date in his journal. Soon he erased the date and wrote it again, aiming to write it even more neatly than before. After a few moments, I asked Jerome to tell me something he knew about the problem, and I encouraged him to record his ideas in his journal. Still, Jerome was reluctant to engage in the problem-solving process.

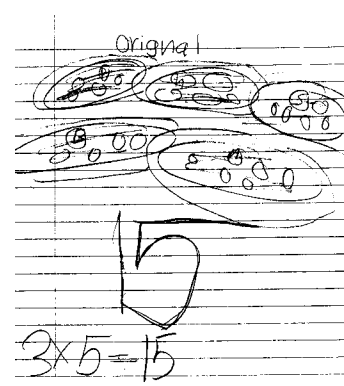
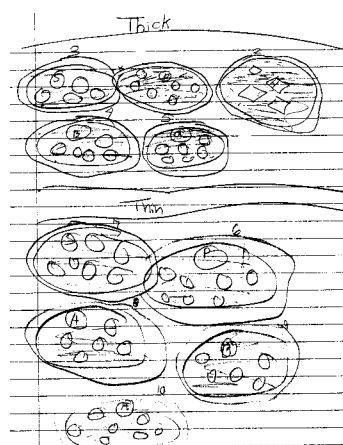
So, how do we teachers proceed in supporting our classes in meeting this first Standard for Mathematical Practice when we have a student or students who, like Jerome, are reluctant to engage in the problem-solving process? To answer this question, we began the school year by immediately engaging students in each of the NCTM Process Standards (2000). Problem solving was at the heart of our efforts as we evolved into a community



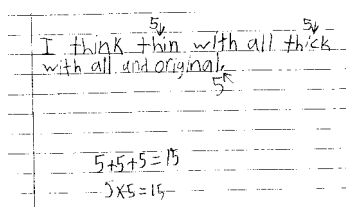
FIGURE 1

Three students' journal responses offered solutions to Tony's Pizza problem.

(a) This student's organized list of the possibilities began with pizza toppings on thick crust, then thin, and then original.



(b) The second student determined a total number of fifteen combinations without considering what the combinations might be.



(c) This formerly reluctant problem solver drew circles to represent toppings and crusts, connecting different combinations with lines. No longer reluctant, he now makes sense of problems and perseveres to solve them.

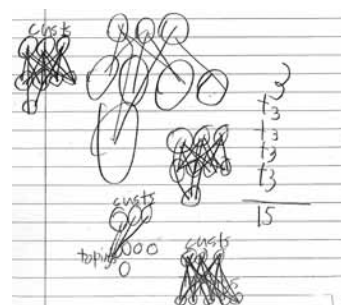
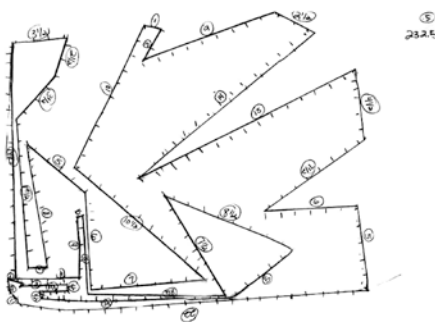


FIGURE 2

Teachers traced Emilio's sample for the Greatest Perimeter task several times to be sure it really was a polygon (assuming he intended the curves to be straight).



engage, often relying on group members to do the work for them or asking the teacher to tell them what they should do. Recognizing that CCSSM indicates we should expect *all* students to “make sense of problems and persevere in solving them” (CCSSI 2010, p. 6), we asked ourselves, What will it take to support our reluctant problem solvers in meeting this very important Standard for Mathematical Practice?

As the school year progressed, we noted that our reluctant problem solvers tended to engage in problem solving more readily when we presented them with what we call *tasks without words*. For these students, finding success with these tasks provided the confidence they needed to move forward in becoming problem solvers. Acknowledging the positive impact that these tasks had on our reluctant problem solvers, we want to support the reader in understanding what tasks without words are, as well as the potential they hold for helping *all* students in meeting the expectations of CCSSM.

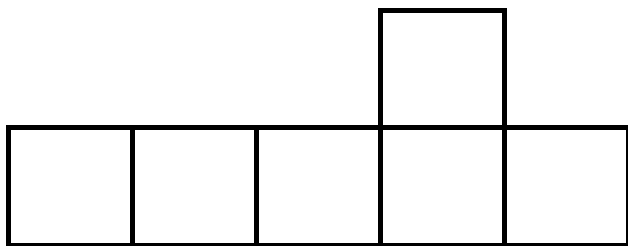
Tasks without words

When we think of problem solving, we often think of tasks, such as the Pizza problem, that require the student to read and make sense of a written passage. In working with the students in our classroom, we noted that several of the reluctant problem solvers were some of our weaker readers. Therefore, we began employing what we refer to as *tasks without words* as a means for engaging them in problem solving. These tasks challenged the students mathematically and engaged them in problem solving. Yet the tasks did not necessarily include words for students to read.

Consider, for example, the Greatest Perimeter task, in which students were challenged to draw a polygon that had the largest perimeter possible and would fit on an 8.5 × 11-inch piece of paper. When completing this task, some students drew a rectangle as close to the edge of the paper as possible, believing it would have the greatest perimeter. As we reminded students that their polygons did not have to be any particular shape, they worked to figure out how to draw a polygon that had a greater perimeter. Students began drawing concave polygons, recognizing that this would most likely create a shape with a greater

FIGURE 3

Ty normally doodled in his journal, but for the Add-a-Square problem (Slovin et al. 2003), he quickly began building the figure and then ardently defended his solution.



of learners. After approximately five weeks, the classroom norms were set. All the students were routinely engaging in the problem-solving process. All the students, that is, except for a small handful who represented the reluctant problem solvers in whom we were interested. Typically, reluctant problem solvers are not as eager to

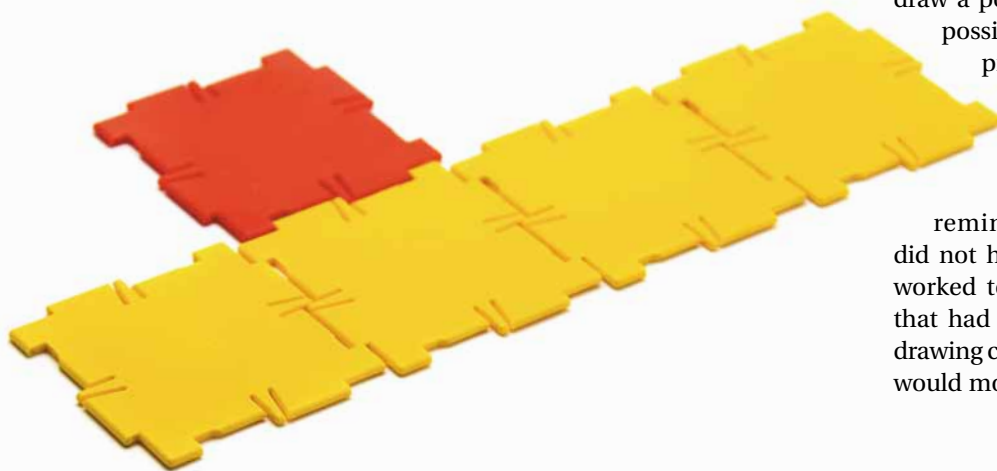


TABLE 1

With slight alterations, the activities in this list of tasks without words could be linked to other domains or grade levels besides those from the Standards for Mathematical Content (CCSSI 2010) that are shown.

| Task | Sample link to the Standards for Mathematical Content | |
|-------------------------|---|--|
| Greatest Perimeter task | 3.MD.8 | Solve real-world and mathematical problems involving perimeters of polygons. . . . |
| Add-a-Square challenge | 3.MD.5a | A square with a side length of 1 unit . . . can be used to measure area. |
| | 3.MD.6 | Measure areas by counting unit squares. . . . |
| | 3.MD.8 | Solve real-world and mathematical problems involving perimeters of problems. |
| Arithmagon problem | 3.NBT.2 | Fluently add and subtract within 1000. . . . |
| Bead problem | 4.OA.5 | Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern. . . . |
| Creating Shapes task | 4.G.3 | Recognize a line of symmetry for a two-dimensional figure. . . . Identify line-symmetric figures. . . . |

perimeter. On any other day, Emilio, one of our reluctant problem solvers, would have watched his group members, allowing time to pass and waiting on someone to say three times, “Emilio, get started.” But this day was different. Emilio eagerly attempted this problem and created a polygon (see **fig. 2**). Clearly, this task without words motivated Emilio to engage in the problem-solving process.

As a second example, consider the Add-a-Square challenge (Slovin et al. 2003). Students were presented with a polygon (see **fig. 3**). Their challenge was to add a square to the figure so that the area increased but the perimeter did not. When Ty received this problem and the accompanying squares with which to work, he quickly began building the figure, attempting to add a square and solve the problem. Once he found his solution, he passionately began justifying to his neighbor how he knew his solution was correct and why his placement of the square did not affect the perimeter. This willingness to engage in problem solving was not typical for Ty, who normally doodled cartoon-like scenes involving dragons and swords in his journal.

Our best response would have been to ask both Emilio and Ty for their thoughts on what features of the tasks caused them to engage as problem solvers. Without having asked them, we are left to consider the features of the two tasks. In class, we routinely used manipulatives, such as rulers and square tiles, which eliminates manipulatives as the feature that appealed to these two reluctant problem solvers. Alternatively, it could have been the mathematics content embedded in the two tasks, namely measurement. Both boys, however, engaged in similar tasks involving other content. This, in turn, led us to believe that the key characteristic that attracted these two reluctant problem solvers was the lack of words in the task.

Samples of tasks without words

Both previous examples of tasks without words involve concepts related to the CCSSM Measurement and Data domain (CCSSI 2010). Such tasks are also available in other domains (see **table 1**). In the following section, we offer a sampling of such tasks. Slightly altering the tasks could link them to other domains or grade levels.

The Creating Shapes task is an example of a problem with multiple entry points, solutions, and extensions.



ANGELA T. BARLOW

The Arithmagon problem

Challenging students to find the “secret numbers” at each vertex of a triangle (see fig. 4), the Arithmagon problem (Levasseur and Cuoco 2003) captures the attention of students and allows them to think logically without having to read a word problem. Secret numbers from two vertices must add to reach the sum recorded on the edge connecting the vertices. You could alter the numbers to challenge students at different grade levels, and students could even develop their own set of numbers for their peers to work with.

The Bead problem

Instead of giving students a set of numbers in a pattern and asking them to decipher the pattern, consider the Bead problem (Krulik and Rudnick 1995), which offers an opportunity to experience patterns presented in a way other than a series of numbers (see fig. 5). To solve the problem, students must determine the pattern so that they can decide how many beads are not showing because they are inside the box. To extend this task, ask students to describe the next section of the chain or to determine how many beads would be on the chain if the pattern were continued to the n th section.

The Creating Shapes task

An activity that has students use pattern blocks to create three different parallelograms, including one with one line of symmetry and one with

no lines of symmetry, the Creating Shapes task offers endless possibilities for extensions. You could easily alter it to create, for example, two different polygons, one with one set of parallel lines and one with one set of perpendicular lines. Alternatively, students could be asked to create polygons with an obtuse angle or an acute angle.

Reflecting on the tasks

As you think about each of these tasks, recognize three key features:

1. Each task engages students in important mathematical ideas.
2. As students approach each task, they are presented with a situation for which they have no immediate route toward a solution. Therefore, they must engage in problem solving.
3. The tasks include few words for the student to read, resulting in a problem that does not look like a “typical” problem.

FIGURE 4

The Arithmagon problem (Levasseur and Cuoco 2003) challenges students without making them read a word problem.

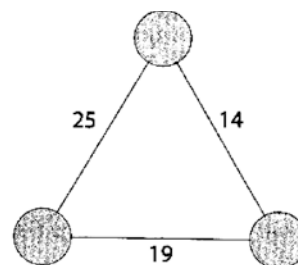
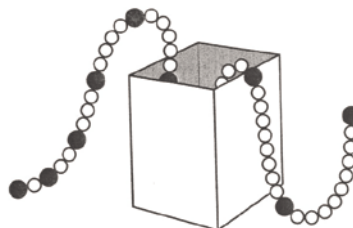


FIGURE 5

To solve the Bead problem (Krulik and Rudnick 1995), students must find the pattern and decide how many beads are inside the box but not visible.



On the basis of our experiences, these tasks without words, as we call them, are attractive to all students, including reluctant problem solvers. We hypothesize that the tasks appeal to reluctant problem solvers for two reasons: (1) The reading requirement is removed, which may interest a student who considers himself or herself to be a struggling reader; and (2) the lack of words gives the task more of a puzzle-like nature, thus appealing to a student's curiosity. In a sense, these tasks are camouflaged problem solving; that is, they engage the student in problem solving without the student knowing.

In *Principles and Standards for School Mathematics*, the authors stated the following:

When challenged with appropriately chosen tasks, students can become confident in their ability to tackle difficult problems, eager to figure things out on their own, flexible in exploring mathematical ideas, and willing to persevere when tasks are challenging. (NCTM 2000, p. 21)

In our experience, tasks without words serve as appropriate tasks for engaging reluctant problem solvers. Regardless of what attracts students to the tasks, they find success with solving the tasks by working to understand the expectations, developing a plan, carrying it out, and being persistent. As these skills develop, they naturally transfer to problems similar to the opening problem. In fact, the student whose work was featured in **figure 1c** was once a reluctant problem solver but is now making sense of problems and persevering to solve them.

As teachers aiming to meet the expectations of CCSSM, we must focus on meeting mathematical content standards while meeting mathematical practice standards. In our opinion, establishing a classroom environment that focuses on engaging students in the NCTM Process Standards (2000) will be sufficient for supporting many students' achievement of the Standards for Mathematical Practice (CCSSI 2010). Some students, however, will be reluctant to engage in problem solving and potentially will fail to meet CCSSM expectations. On the basis of our experiences, though, tasks without words hold the potential for engaging reluctant problem solvers

in the problem-solving process and for serving to jump-start them toward achieving the Standards.

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